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NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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[Continued from January Number.]

PROPOSITION XXXII. *Now I say there is (in hypothesis of acute angle) a certain determinate acute angle BAX drawn under which AX* (Fig. 33.) *only at an infinite distance meets BX, and thus is a limit in part from within, in part from without; on the one hand of all those which under lesser acute angles meet the aforesaid BX at a finite distance; on the other hand also of the others which under greater acute angles, even to a right angle inclusive, have a common perpendicular in two distinct points with BX.*

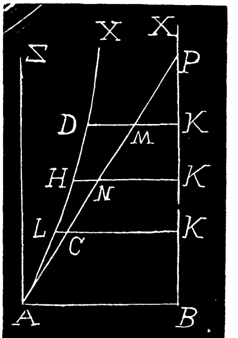

Fig. 33.

PROOF. First it holds (from Cor. II. after Proposition XXIX.) that no determinate acute angle will be the greatest of all drawn under which a straight from the point A meets the aforesaid BX at a finite distance

Secondly, it holds in like manner that (in hypothesis of acute angle) no acute angle will be the least of all drawn under which a straight has a common perpendicular in two distinct points with BX; since indeed (from what precedes) there can be no determinate limit, such that there cannot be found, under a lesser angle

constituted at the point A, a common perpendicular in two distinct points, which is less than any assignable length R.

And hence follows thirdly, that (in this hypothesis) there must be a certain determinate acute angle BAX, drawn under which AX so approaches ever more to BX, that only at an infinite distance does it meet it.

But further that this AX is a limit in part from within in part from without of each of the aforesaid classes of straights is proved thus. First, it agrees with those straights which meet BX at a finite distance since it also finally meets; but it differs, because it meets only at an infinite distance.

But secondly it also agrees, and at the same time differs from those straights which have a common perpendicular in two distinct points with BX; because it also has a common perpendicular with BX; but in one and the same point X infinitely distant. But this latter ought to be considered demonstrated in Proposition XXVIII., as I point out in its corollary.

Therefore it holds, that (in the hypothesis of acute angle) there will be a certain determinate acute angle BAX, drawn under which AX only at an infinite distance meets BX, and thus is a limit in part from within, in part from without; on the one hand of all those which under lesser acute angles meet the aforesaid BX at a finite distance; on the other hand also of the others which under greater acute angles, even to a right angle inclusive, have a common perpendicular in two distinct points with BX. Quod erat etc.

[To be Continued.]

ON THE BEST METHOD OF SOLVING THE MARKINGS OF JUDGES OF CONTESTS.

By F. R. MOULTON.

The fact that many different methods are in use for deciding from the markings of judges the relative standing of the participants in oratorical and similar contests, and that several different methods have been stated to be the best by Professors of Mathematics in some of our colleges, may be taken as the excuse for this paper. It is questionable whether such a problem can be solved by perfectly rigorous mathematical processes, but it has seemed that a method similar to that of Hagen in the theory of probability may be applied with advantage. Let us agree to adopt the following hypotheses.

(1) The judges mark each contestant independently and by the same scale.

(2) There is a true marking for each contestant as compared to a fixed ideal.

(3) The deviation of each judge's markings from these true markings, are the result of a very great many influences, such as, the inherited inclinations re-